Transport through interacting Majorana devices

Reinhold Egger
Institut für Theoretische Physik
Overview

Coulomb charging effects on quantum transport through Majorana nanowires:

- Two-terminal device: Majorana single-charge transistor
  
  Zazunov, Levy Yeyati & Egger, PRB 84, 165440 (2011)
  Hützen, Zazunov, Braunecker, Levy Yeyati & Egger, PRL 109, 166403 (2012)

- Multi-terminal device: 'Topological‘ Kondo effect with stable non-Fermi liquid behavior
  
  With interactions in the leads: novel unstable fixed point
  
  Altland & Egger, PRL 110, 196401 (2013)
  Zazunov, Altland & Egger, NJP 16, 015010 (2014)

  'Majorana spin‘ dynamics near strong coupling
  
  Altland, Beri, Egger & Tsvelik, arXiv:1312.3802

  Non-Fermi liquid manifold: coupling to bulk superconductor
  
Majorana bound states

- Majorana fermions
  - Non-Abelian exchange statistics \( \gamma_j = \gamma^+_j \) \( \{ \gamma_i, \gamma_j \} = 2\delta_{ij} \)
  - Two Majoranas = nonlocal fermion \( d = \gamma_1 + i\gamma_2 \)
  - Occupation of single Majorana ill-defined: \( \gamma^+\gamma = \gamma^2 = 1 \)
  - Count state of Majorana pair \( d^+d = 0,1 \)
- Realizable (for example) as end states of spinless 1D p-wave superconductor (Kitaev chain)
  - Recipe: Proximity coupling of 1D helical wire to s-wave superconductor
  - For long wires: Majorana bound states are zero energy modes

References:
Experimental Majorana signatures

InSb nanowires expected to host Majoranas due to interplay of
- strong Rashba spin orbit field
- magnetic Zeeman field
- proximity-induced pairing

Oreg, Refael & von Oppen, PRL 2010
Lutchyn, Sau & Das Sarma, PRL 2010

Transport signature of Majoranas:
Zero-bias conductance peak due to resonant Andreev reflection

Bolech & Demler, PRL 2007
Law, Lee & Ng, PRL 2009
Flensberg, PRB 2010

See also: Rokhinson et al., Nat. Phys. 2012;
Churchill et al., PRB 2013
Zero-bias conductance peak

Possible explanations:
- Majorana state (most likely!)
- Disorder-induced peak
- Smooth confinement
- Kondo effect

Bagrets & Altland, PRL 2012
Kells, Meidan & Brouwer, PRB 2012
Lee et al., PRL 2012

Mourik et al., Science 2012
Suppose that Majorana mode is realized...

- Quantum transport features beyond zero-bias anomaly peak? Coulomb interaction effects?
- Simplest case: Majorana single charge transistor
  - 'Overhanging' helical wire parts serve as normal-conducting leads
  - Nanowire part coupled to superconductor hosts pair of Majorana bound states
  - Include charging energy of this 'dot'
Majorana single charge transistor

- Floating superconducting 'dot' contains two Majorana bound states tunnel-coupled to normal-conducting leads
- Charging energy finite

Consider universal regime:
- Long superconducting wire: Direct tunnel coupling between left and right Majorana modes is assumed negligible
- No quasi-particle excitations: Proximity-induced gap is largest energy scale of interest

Hützen et al., PRL 2012
Hamiltonian: charging term

- Majorana pair: nonlocal fermion \( d = \gamma_L + i\gamma_R \)
- Condensate gives another zero mode
  - Cooper pair number \( N_c \), conjugate phase \( \phi \)
- Dot Hamiltonian (gate parameter \( n_g \))

\[
H_c = E_c \left( 2N_c + d^+ d - n_g \right)^2
\]

Majorana fermions couple to Cooper pairs through the charging energy
Tunneling

- Normal-conducting leads: noninteracting fermions (effectively spinless helical wire)
  - Applied bias voltage $V = \text{chemical potential difference}$
- Tunneling of electrons from lead to dot:
  - Project electron operator in superconducting wire part to Majorana sector
  - Spin structure of Majorana state encoded in tunneling matrix elements

*Flensberg, PRB 2010*
Tunneling Hamiltonian

Source (drain) couples to left (right) Majorana only:

\[ H_t = \sum_{j=L,R} t_j c_j^+ \eta_j + h.c. \]

- \( \eta_j = \left( d \pm e^{-i\phi} d^+ \right)/2 \)
- Respects current conservation
- Hybridizations: \( \Gamma_{L/R} \sim \rho_0 |t_{L/R}|^2 \)

**Normal tunneling** \( \sim c^+ d, d^+ c \)
- Either destroy or create nonlocal d fermion
- Condensate not involved

**Anomalous tunneling** \( \sim c^+ e^{-i\phi} d^+, de^{i\phi} c \)
- Create (destroy) both lead and d fermion
  & split (add) a Cooper pair
Absence of even-odd effect

- Without Majorana states: Even-odd effect
- With Majorana states: no even-odd effect!
- Tuning wire parameters into the topological phase removes even-odd effect

picture from: Fu, PRL 2010
Majorana Meir-Wingreen formula

- Exact expression for interacting Majorana dot
  \[ I_{j=L,R} = \frac{e\Gamma_j}{\hbar} \int d\varepsilon \ F(\varepsilon - \mu_j) \text{Im} \ G_{\eta_j}^{\text{ret}}(\varepsilon) \]

- Lead Fermi distribution encoded in \( F(\varepsilon) = \tanh(\varepsilon/2T) \)

- Proof uses \( \eta^+_j \eta_j = 1 \)

- Differential conductance:
  \[ G = \frac{dI}{dV} \]

  \[ I = \frac{(I_L - I_R)}{2} \]

  Here: symmetric case \( \Gamma_L = \Gamma_R = \Gamma/2 \)
Noninteracting case: Resonant Andreev reflection

- $E_c = 0$ Majorana spectral function
  \[ -\text{Im} \, G_{\gamma_j}^{\text{ret}}(\varepsilon) = \frac{\Gamma_j}{\varepsilon^2 + \Gamma_j^2} \]

- $T = 0$ differential conductance:
  \[ G(V) = \frac{2e^2}{h} \frac{1}{1 + (eV/\Gamma)^2} \]

- Currents $I_L$ and $I_R$ fluctuate independently; superconductor is effectively grounded

- Perfect Andreev reflection via Majorana state
  - Zero-energy Majorana bound state leaks into lead
Strong blockade: Electron teleportation

- Peak conductance for half-integer $n_g$
- Strong charging energy then allows only two degenerate charge configurations
- Model maps to spinless resonant tunneling model
- Linear conductance (T=0): $G = \frac{e^2}{h}$
- Interpretation: Electron teleportation due to nonlocality of d fermion

$Fu$, *PRL* 2010
Crossover from resonant Andreev reflection to electron teleportation

- Keldysh approach yields full action in phase representation
  
  Zazunov, Levy Yeyati & Egger, PRB 2011

- Practically useful in weak Coulomb blockade regime: interaction corrections to conductance

- Full crossover from three other methods:
  
  Hützen et al., PRL 2012

- **Master equation** for T>Γ: include sequential and all cotunneling processes (incl. local and crossed Andreev reflection)

- **Equation of motion approach** for peak conductance

- **Zero bandwidth model** for leads: exact solution
Coulomb oscillations

Valley conductance dominated by elastic cotunneling

Master equation

\[ T = 2\Gamma \]
Peak conductance: from resonant Andreev reflection to teleportation
Finite bias sidepeaks

![Graph showing the dependence of the conductance on energy for different values of $E_c$. The graph includes a legend for $n_g = 1/2$ and $n_g = 1$. The master equation is $\Gamma = \frac{2}{T}$.](image-url)
Finite bias sidepeaks

- On resonance: sidepeaks at $eV = 4nE_c$
- $\mu_{L,R}$ resonant with two (almost) degenerate higher order charge states: additional sequential tunneling contributions
- Requires change of Cooper pair number - only possible through anomalous tunneling: without Majoranas no side-peaks
- Similar sidepeaks away from resonance
Multi-terminal case

- Now $N>1$ helical wires: $M$ Majoranas modes, tunnel-coupled to helical Luttinger liquid wires, $g \leq 1$
- Bosonization of leads: **Klein-Majorana fusion**
  - Klein factors $\rightarrow$ additional Majorana fermion for each lead
  - Combine Klein-Majorana and 'true' Majorana at each contact to build auxiliary fermion $f_j$
  - All occupation numbers $f_j^+f_j$ are conserved and can be gauged away: purely bosonic problem remains!

Zazunov, Altland & Egger, NJP 2014
Altland & Egger, PRL 2013
Beri & Cooper, PRL 2012; Beri, PRL 2013
Charging effects: dipole confinement

- High energy scales $> E_c$: charging effects irrelevant
  - Electron tunneling amplitudes from lead j to dot renormalize independently upwards
    \[ t_j(E) \sim E^{-1 + \frac{1}{2g}} \]

- RG flow towards resonant Andreev reflection fixed point

- For $E < E_c$: charging induces 'confinement'
  - In- and out-tunneling events are bound to 'dipoles' with coupling $\lambda_{j \neq k}$: entanglement of different leads
  - Dipole coupling describes amplitude for 'teleportation' from lead j to lead k
  - 'Bare' value
    \[ \lambda^{(1)}_{jk} = \frac{t_j(E_c) t_k(E_c)}{E_c} \sim E_c^{-3 + \frac{1}{g}} \quad \text{large for small } E_c \]
RG equations in dipole phase

- Energy scales below $E_c$: effective phase action
  \[ S = S_{Lutt}[\Phi] - \sum_{j \neq k} \lambda_{jk} \int d\tau \cos(\Phi_j - \Phi_k) \]

- One-loop RG equations
  \[ \frac{d\lambda_{jk}}{dl} = - (g^{-1} - 1)\lambda_{jk} + \nu \sum_{m \neq (j,k)}^M \lambda_{jm}\lambda_{mk} \]

- Suppression by Luttinger tunneling DoS
- Enhancement by dipole fusion processes

- RG-unstable intermediate fixed point with isotropic couplings (for $M > 2$ leads)
  \[ \lambda_{j \neq k} = \lambda^* = \frac{g^{-1} - 1}{M - 2} \nu \]
- RG flow towards strong coupling for \( \langle \lambda^{(1)} \rangle > \lambda^* \)
  - Always happens for moderate charging energy
- Flow towards isotropic couplings: anisotropies are RG irrelevant
- Perturbative RG fails below Kondo temperature
  \[
  T_K \approx E_c e^{-\lambda^* / \langle \lambda^{(1)} \rangle}
  \]
Topological Kondo effect

- Refermionize for $g=1$:
  \[ H = -i \int d\gamma \sum_{j=1}^{M} \Psi_j^+ \partial_x \Psi_j + i \lambda \sum_{j \neq k} \Psi_j^+ (0) S_{jk} \Psi_k (0) \]

- Majorana bilinears
  \[ S_{jk} = i \gamma_j \gamma_k \]

- 'Reality' condition: SO(M) symmetry [instead of SU(2)]
- Nonlocal realization of 'quantum impurity spin'
- Nonlocality ensures stability of Kondo fixed point

- Real Majorana basis
  \[ \Psi(x) = \mu + i \xi \]
  for leads: SO$_2$(M) Kondo model

\[ H = -i \int d\gamma \mu^T \partial_x \mu + i \lambda \mu^T (0) S \mu (0) + [\mu \leftrightarrow \xi] \]
Transport properties near unitary limit

- Temperature & voltages $< T_K$:
  - Dual instanton version of action applies near strong coupling limit
  - Nonequilibrium Keldysh formulation

- Linear conductance tensor

$$G_{jk} = e \frac{\partial I_j}{\partial \mu_k} = \frac{2e^2}{\hbar} \left( 1 - \left( \frac{T}{T_K} \right)^{2y-2} \right) \left[ \delta_{jk} - \frac{1}{M} \right]$$

- Non-integer scaling dimension $y = 2g \left( 1 - \frac{1}{M} \right) > 1$
  - implies non-Fermi liquid behavior even for $g=1$

- completely isotropic multi-terminal junction
Fano factor

- Backscattering correction to current near unitary limit for $\sum_j \mu_j = 0$

$$\delta I_j = -\frac{e}{\hbar} \sum_k \left| \frac{\mu_k}{T_K} \right|^{2y-2} \left( \delta_{jk} - \frac{1}{M} \right) \mu_k$$

- Shot noise:

$$\tilde{S}_{jk}(\omega \to 0) = \int dt \ e^{i\omega t} \left( \langle I_j(t)I_k(0) \rangle - \langle I_j \rangle \langle I_k \rangle \right)$$

$$\tilde{S}_{jk} = -\frac{2ge^2}{\hbar} \sum_l \left( \delta_{jl} - \frac{1}{M} \right) \left( \delta_{kl} - \frac{1}{M} \right) \left| \frac{\mu_l}{T_K} \right|^{2y-2} |\mu_l|$$

- Universal Fano factor, but different value than for SU(N) Kondo effect

Sela et al. PRL 2006; Mora et al., PRB 2009
Majorana spin dynamics

Altland, Beri, Egger & Tsvelik, arXiv:1312.3802

- Overscreened multi-channel Kondo fixed point: massively entangled effective impurity degree remains at strong coupling: "Majorana spin"
- Probe and manipulate by coupling of Majoranas

\[ H_Z = \sum_{jk} h_{jk} S_{jk} \]

- 'Zeeman fields' \( h_{jk} = -h_{kj} \): overlap of Majorana wavefunctions within same nanowire
- Couple to \( S_{jk} = i\gamma_j\gamma_k \)
Majorana spin near strong coupling

Bosonized form of Majorana spin at Kondo fixed point:
\[ S_{jk} = i\gamma_j\gamma_k \cos[\Theta_j(0) - \Theta_k(0)] \]

- Dual boson fields \( \Theta_j(x) \) describe 'charge' (not 'phase') in respective lead
- Scaling dimension \( \gamma_Z = 1 - \frac{2}{M} \) \( \rightarrow \) RG relevant
- Zeeman field ultimately destroys Kondo fixed point & breaks emergent time reversal symmetry
- Perturbative treatment possible for \( T_h < T < T_K \)

\[
T_h = \left( \frac{h_{12}}{T_K} \right)^{M/2} T_K
\]

dominant 1-2 Zeeman coupling:
Crossover $\text{SO}(M) \rightarrow \text{SO}(M-2)$

- Lowering $T$ below $T_h$ → crossover to another Kondo model with $\text{SO}(M-2)$ (Fermi liquid for $M<5$)
- Zeeman coupling $h_{12}$ flows to strong coupling → $\gamma_1, \gamma_2$ disappear from low-energy sector
- Same scenario follows from Bethe ansatz solution
  
  Altland, Beri, Egger & Tsvelik, arXiv:1403.0113

- Observable in conductance & in thermodynamic properties
**SO(M)→SO(M-2): conductance scaling**

for single Zeeman component $h_{12} \neq 0$ consider $G_{jj} \ (j \neq 1,2)$

(diagonal element of conductance tensor)
Multi-point correlations

- Majorana spin has nontrivial multi-point correlations at Kondo fixed point, e.g. for $M=3$ (absent for SU(N) case!)

\[ \left\langle T_\tau s_j(\tau_1) s_k(\tau_2) s_l(\tau_3) \right\rangle \sim \frac{\epsilon_{jkl}}{T_K(\tau_{12}\tau_{13}\tau_{23})^{1/3}} \quad s_j = \epsilon_{jkl} S_{kl} \]

- Observable consequences for time-dependent 'Zeeman' field $B_j = \epsilon_{jkl} h_{kl}$ with $\vec{B}(t) = (B_1 \cos(\omega_1 t), B_2 \cos(\omega_2 t), 0)$
  - Time-dependent gate voltage modulation of tunnel couplings
  - Measurement of 'magnetization' by known read-out methods
- Nonlinear frequency mixing
  \[ \left\langle s_3(t) \right\rangle \sim B_1 B_2 \cos[ (\omega_1 \pm \omega_2) t ] \]
- Oscillatory transverse spin correlations (for $B_2 = 0$)
  \[ \left\langle s_2(t) s_3(0) \right\rangle \sim B_1 \frac{\cos(\omega_1 t)}{(\omega_1 t)^{2/3}} \]
Conclusions

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  With electron-electron interactions in the leads: novel unstable fixed point
  
  Altland & Egger, PRL 110, 196401 (2013)
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- Dynamics of ‘impurity spin‘ in strong coupling regime
  
  Altland, Beri, Egger & Tsvelik, arXiv:1312.3802

- Non-Fermi liquid manifold: coupling to bulk superconductor

THANK YOU FOR THE ATTENTION!